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# The pion form factor within the hidden local symmetry model

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**Abstract.** We analyze a pion form factor formulation which fulfills the Analyticity requirement within the Hidden Local Symmetry (HLS) Model. This implies an $s$–dependent dressing of the $\rho-\gamma$ VMD coupling and an account of several coupled channels. The corresponding function $F_\pi(s)$ provides nice fits of the pion form factor data from $s=-0.25$ to $s=1$ GeV$^2$. It is shown that the coupling to $K\overline{K}$ has little effect, while $\omega\pi^0$ improves significantly the fit probability below the $\phi$ mass. No need for additional states like $\rho(1450)$ shows up in this invariant–mass range. All parameters, except for the subtraction polynomial coefficients, are fixed from the rest of the HLS phenomenology. The fits show consistency with the expected behaviour of $F_\pi(s)$ at $s=0$ up to $\mathcal{O}(s^2)$ and with the phase shift data on $\delta_1^1(s)$ from threshold to somewhat above the $\phi$ mass. The $\omega$ sector is also examined in relation with recent data from CMD–2.

## 1 Introduction

In the physics of exclusive processes, the pion form factor $F_\pi(s)$ plays an important role. It is indeed a fundamental tool in order to estimate precisely the hadronic contribution to the muon anomalous magnetic moment (for recent works, see [1] and [2] where an exhaustive list of references can be found). It is also an important information, as it allows to test the predictions of Chiral Perturbation Theory (ChPT) which describes the behaviour of QCD at low energies where non–perturbative effects dominate. Among very recent works on this classical subject, let us quote [1, 3, 4].

Several descriptions of the pion form factor are proposed. For instance, [1] gives a parametrization of the $P-$wave $\pi\pi$ phase shift $\delta_1^1(s)$ derived from general analyticity principles supplemented with some properties related with the existence of the $\rho^0(770)$ meson. Watson theorem relates $F_\pi(s)$ with the $\pi\pi$ phase shift by proving that $\mathrm{Arg}[\mathrm{F_\pi(s)}]=\delta_1^1(\mathrm{s})$ up to the first inelastic threshold. In principle, this is located at the four–pion threshold, however experimental data [5], especially on $P$–wave inelasticity, show that $\delta_1^1(s)$ can be considered elastic with a nice precision up to the 0.95 GeV region. The free parameters of the function defined by [1] are fitted on Aleph [6] and Opal [7] $\tau$ decay data on the two–pion final state. The derived phase [1] is shown to predict impressively the phase of [8]. In this approach, the role of the $\rho(770)$ meson is obvious; what is less obvious is whether additional states like the $\rho(1450)$ play any role below $\sqrt{s}=1$ GeV. Actually, while focussing on estimating hadronic contributions to the muon anomalous magnetic moment, it is not a real concern.

In the same spirit, [3] starts from phase shift data [5] measured up to $\sqrt{s}\simeq 2$ GeV, assumes Watson theorem and fit the Aleph [6] and CLEOII [9] relevant data sets with:

$$F_\pi(s)=\exp\left\{\alpha_1 s+\frac{1}{2}\alpha_2 s^2+\frac{s^3}{\pi}\int_{4m_\pi^2}^{\Lambda^2}\frac{dz}{z^3}\frac{\delta_1^1(z)}{z-s-i\epsilon}\right\}$$

where $\Lambda$ is some cut–off and $\alpha_1$ and $\alpha_2$ are free parameters.

The approach of [4] relies instead on the Resonance Chiral Theory developed in [10], where vector mesons are explicitly introduced in the Lagrangian. Here the parameters to be fitted are the masses and couplings associated with the usual vector meson nonet (those containing the $\rho(770)$) and the one associated with the $\rho(1450)$ meson. Focusing on the $\rho(770)$ nonet, this mass is fit as $M_{V_1}\simeq 840$ MeV, which does not prevent the Breit–Wigner $\rho(770)$ parameters derived from this fit [4] to be very close to expectations [11]. Here again, the phase predicted from fits to $|F_\pi(s)|^2$ can be compared to data [5] and an effect attributed to the $\rho(1450)$ meson seems to affect somewhat the phase shift around $s=1$ GeV$^2$.

Beside these approaches, the most usual framework is VMD in which $F_\pi(s)$ is represented as a sum of vector meson contributions; traditionally, these are chosen as Gounaris–Sakurai functions [12]. Focussing on $e^+e^-$ annihilations, this is illustrated by the reference fit in [13] to the data collected by the OLYA,CMD and DM1 Collaborations [13,14]. The data set recently collected by CMD–2 [15] is also fitted in this way. In this last study, two prominent conclusions show up: the $\omega\to\pi\pi$ branching fraction is found smaller than previously measured [13] ($1.33\pm0.25\%$ instead of $2.21\pm0.30\%$) and a contribution from the $\rho(1450)$ meson is needed in order to reach a good description of the data set (fully located below 1 GeV).

Recently, it has been remarked [16] that the Hidden Local Symmetry (HLS) Model [17] provides another consistent framework for data analysis and a new expression for $F_\pi(s)$ at low energies. Indeed, besides the usual vector meson exchanges, this model predicts that some departure from standard VMD could show up as a residual direct coupling $\gamma\pi^+\pi^-$. The form factor written[1]:

$$F_\pi(s) = 1 - \frac{a}{2} - \frac{f_{\rho\gamma} g_{\rho\pi\pi}}{s - m_\rho^2 + i m_\rho \Gamma_\rho(s)} - \frac{f_{\omega\gamma} e^{i\phi} g_{\omega\pi\pi}}{s - m_\omega^2 + i m_\omega \Gamma_\omega(s)} \tag{1}$$

has been used to fit the data then available [13,14]. This expression provided a nice fit [16] for the whole energy range below $s \leq 1$ GeV$^2$ without introducing any additional vector state like the $\rho(1450)$ meson. For the HLS parameter $a$, the fit returned $a = 2.36 \pm 0.02$ in contrast with standard VMD where $a = 2$.

This HLS based model has been used, besides the usual Gounaris–Sakurai formula, to fit the recent CMD–2 data set and has been found to provide as good results [15]. In this case, the fit returned $a = 2.336 \pm 0.015_{\rm stat} \pm 0.007_{\rm syst}$, in obvious correspondence with the previous estimate derived from fit [16] to the former $e^+e^-$ data sets [13,14]. As for the previous data sets, when using the HLS model as expressed by (1), no effect below the $\phi$ mass was observed which could be attributed to a $\rho(1450)$ contribution in contrast with the standard (VMD) fit [15].

The use of a varying width Breit–Wigner formula to account for the $\rho$ propagator, as done in (1), permits a convenient description of the pion form factor from threshold to the $\phi(1020)$ mass. However, this approximation prevents from drawing any conclusion on the pion form factor behaviour below the 2–pion threshold, as this expression for the pion form factor (which is *not* an analytic function) cannot be analytically continued in this region – and then in the neighborhood of $s = 0$. However, the condition $F_\pi(0) = 1$ can instead be propagated to the 2–pion threshold using known information from ChPT; this predicts $F_\pi(4m_\pi^2) = 1.17 \pm 0.01$ to be compared with the fit result [16] $F_\pi(4m_\pi^2) = 1.176 \pm 0.001$. Moreover, the phase of $F_\pi(s)$ resulting from the fit performed with (1) is a prediction for the $\delta_1^1(s)$ $\pi\pi$ phase shift and also compares well [16] with the phase shift data of [19]. Even if this comparison sounds successful, it is, however, desirable to avoid such an indirect consistency check by using from start an expression of the pion form factor in terms of analytic functions and constrained in order to satisfy $F_\pi(0) = 1$.

The aim of the present paper is to examine the pion form factor in the context of the HLS Model by taking into account both the non–anomalous [17] and anomalous [20] sectors. This extends the previous study performed in [16] by carefully considering the Analyticity requirement and by examining in detail the effect of having several channels coupled to $\pi\pi$ within the HLS Model. This study will be done by imposing the pion factor to fulfill the numerical constraints derived from other sectors of low energy phenomenology accessible to the HLS Model. Indeed, as shown in our previous studies [21–23], anomalous decays of the kind $V \to \gamma P$ or $P \to \gamma V$ and leptonic decays ($V \to e^+e^-$) fix with a valuable precision the basic parameters of the HLS Model ($a$, $g$) beside the breaking parameters. Whether these constraints are well accepted by the data on the pion form factor is indeed an important issue for test.

Loop effects cannot be avoided in problems where the $\rho$ meson plays a crucial role. These will be considered in the framework of the one–loop order treatment proposed in [22]. Doing this way, one limits the possible couplings by neglecting intermediate states with more than two particles which generate multiparticle loops; these are expected to produce small effects [4]. This is supported by the experimental data of [5], which exhibit a $\pi\pi$ $P$–wave elasticity consistent with 1 up to the $\phi$ mass and even slightly above.

In Sect. 2, we derive the pion form factor $F_\pi(s)$ in accordance with Analyticity; we show how the $\rho$ propagator has to be dressed and that the $\gamma - \rho$ coupling becomes invariant–mass dependent at the same order. In Sect. 3, we examine the loop corrections and show that choosing the subtraction polynomial coefficients as fit parameters is consistent. For clarity, we refer only to the non–anomalous sector of the HLS Model in the body of the text; in order to deal with the anomalous sector, more information is given in the Appendices.

In Sect. 4, we recall the results obtained elsewhere concerning the HLS phenomenology, which are imposed as constraints when fitting the pion form factor. It should be noted that our $F_\pi(s)$ has to be consistent with the $\rho$ mass derived from the HLS–KSFR relation (827 MeV). In Sect. 5, we remind which kind of information can act as (external) probes for our HLS Model: the $\pi\pi$ phase shift $\delta_1^1(s)$ and the (polynomial) behaviour of $F_\pi(s)$ near $s = 0$. Our fit strategies and results on the pion form factor are the purpose of Sect. 6, while the short Sect. 7 summarizes our fit results concerning the $\omega$ contribution, especially $\mathrm{Br}(\omega \to \pi\pi)$. Finally, Sect. 8 is devoted to conclusions. In three Appendices, we outline the Lagrangian content of our model and the loop structure; we also give some information about all coupling constants relevant for the pion form factor within the HLS Model.

[1] We use the so–called Orsay Phase formulation for the isospin breaking term commented on below. It has been shown in [18] that the $\omega$ term in (1) actually approximates an analytic function which vanishes at $s = 0$ and then does not affect the condition $F_\pi(0) = 1$. On the other hand, as will be recalled in Sect. 2, the coupling constants and the $\rho$ Higgs–Kibble mass are such that $f_{\rho\gamma} g_{\rho\pi\pi}/m_\rho^2 \equiv a/2$. Therefore, $F_\pi(0) = 1$ is certainly fulfilled by (1), up to ($\rho$) finite width effects. If instead of $\Gamma_\rho(s)$ in (1) one introduces the finite width effects by $\Theta(s - 4m_\pi^2)\Gamma_\rho(s)$, where $\Theta(s - 4m_\pi^2)$ is the usual step function, then $F_\pi(0) = 1$ is always fulfilled; we show below that a consistent treatment of loop effects is equivalent to introducing this step function

## 2 The pion form factor in the hls model

The pion form factor is derived from the Lagrangian given in Appendix A. This contains the traditional pieces [17] of the HLS Lagrangian and includes the SU(3) symmetry breaking procedure as defined in [24]. Nonet symmetry breaking is also introduced in the way discussed in [23]; as will be seen below, this last breaking plays some role, however minor if only the pion form factor is concerned. It also contains an anomalous ($VVP$) piece from the FKTUY Lagrangian [20] broken in the way described in [21, 22]; the $\omega\rho\pi$ term of this Lagrangian (which is not affected by symmetry breaking effects) plays an important role in our formulation of the pion form factor; the role played by the other anomalous couplings will be shown to be marginal.

Actually, what comes out of the non–anomalous sector of the HLS Model [17] at tree level is simply (1) without the $\rho$ width term and amputated from the $\omega$ contribution which corresponds to some breaking of Isospin Symmetry. Omitting these terms, (1) obviously meets the Analyticity requirement (actually, it defines a meromorphic function) but is of little use to describe real data from threshold to the $\phi$ mass. Indeed, the $\rho$ propagator which actually occurs there is the bare propagator $D_0(s) = (s - m_\rho^2)^{-1}$ which exhibits a pole on the physical region $s \geq 4m_\pi^2$.

Introducing one–loop effects modifies the picture. First, dressing of the $\rho$ bare propagator moves the singularities outside the physical region. Second, loop effects modify in a $s$–dependent way the transition amplitude $\gamma \to \rho$ as was first shown in [25]. In the non–anomalous HLS Lagrangian, this $s$–dependence is already generated by the departure of the parameter $a$ from its traditional VMD value ($a = 2$); this can be seen directly from the coupling expressions given in Appendix C. SU(3) Symmetry breaking and anomalous couplings both introduce further $s$–dependent contributions. As global fits to radiative and leptonic decay widths favors $a \simeq 2.5$ [21,22], this $s$–dependence of the $\gamma \to \rho$ transition amplitude has certainly to be considered.

At one–loop order, the dressed propagator $D(s)$ is given by the Schwinger–Dyson Equation, which writes:

$$D^{-1}(s) = D_0^{-1}(s) - \Pi_{\rho\rho}(s) \quad (2)$$

where $\Pi_{\rho\rho}$ is the $\rho$ self–energy, the content of which being examined in some detail in Appendix B. Within the non–anomalous HLS Model [17], contributions to the $\rho$ self–energy come only from pion and kaon loops; if one considers also the anomalous sector of the HLS model, the (FKTUY) Lagrangian of [20], additional $VP$ loops have to be introduced, especially $\omega\pi^0$ which threshold is lower in mass than $K\overline{K}$. This is outlined in Appendix B.

It is expected that the correct expression for the isospin 1 part of the pion form factor is obtained by replacing the denominator in (1) by the dressed propagator $D(s)$ just defined. This can be derived by resumming formally an obvious infinite series of terms, each containing bare propagators and loops (referred to in [4] as Dyson–Schwinger Summation). The same final expression can also be obtained more directly by adding an effective piece [22] to the HLS Lagrangian of the form $\Pi_{\rho\rho}(s)\rho^2/2$, which turns out to modify the vector meson mass term by a $s-$dependent piece. The (dressed) $\rho$ propagator is then derived from this effective Lagrangian at tree level. This method has been discussed in [22].

When breaking Isospin Symmetry within the HLS Model, charged and neutral kaons carry different masses and this generates a $\rho-\omega$ mass–dependent transition term [18]. It was shown in [18] that this gives rise to an $\omega$ contribution to the pion form factor which approximates naturally in the form shown in (1), precisely. It was also shown [18] that the proposed way of breaking Isospin Symmetry makes the $\omega$ contribution vanishing at $s = 0$ and thus does not affect the $F_\pi(0) = 1$ condition.

As stated above, considering one–loop corrections also modifies the $\gamma \to \rho$ transition amplitude in a $s$–dependent way. This can be derived directly using standard Feynman rules and turns out to modify the original $\rho\gamma$ coupling in the following way [25]:

$$-e\ f_{\rho\gamma} \Rightarrow -e\ [f_{\rho\gamma} - \Pi_{\rho\gamma}(s)] \quad (3)$$

We recall (see the Appendices) that the universal vector coupling $g$ is related to $g_{\rho\pi\pi}$ by $g_{\rho\pi\pi} = ag/2$ and that an (extended) KSFR relation holds within the HLS model $m_\rho^2 = ag^2 f_\pi^2$. We also have $f_{\rho\gamma} = m_\rho^2/g$. In order to write (3), a factor of $e$ has been extracted from $\Pi_{\rho\gamma}(s)$ which is thus of order $g$ in couplings.

The content of $\Pi_{\rho\rho}(s)$ and $\Pi_{\rho\gamma}(s)$ is the purpose of Sect. 3 and of Appendix B, where this is discussed in some detail.

Therefore, taking into account one–loop corrections, the isospin 1 part of the pion form factor is:

$$F_\pi(s) = 1 - \frac{a}{2} - \frac{[f_{\rho\gamma} - \Pi_{\rho\gamma}(s)]\, g_{\rho\pi\pi}}{s - m_\rho^2 - \Pi_{\rho\rho}(s)}. \quad (4)$$

The non-anomalous sector of the HLS Model contributes to $\Pi_{\rho\gamma}(s)$ and $\Pi_{\rho\rho}(s)$ through pion and kaon loops weighted by the appropriate coupling constant combinations. The anomalous (FKTUY) part of the Lagrangian provides additional $VP$ loops (see Appendix B). The properties of these loop corrections are discussed in detail in the next section.

By using the expressions for $g_{\rho\pi\pi}$, $f_{\rho\gamma}$ and $m_\rho^2$ recalled just above, one can check that $F_\pi(0) = 1$ is fulfilled if and only if:

$$\Pi_{\rho\rho}(0) + g\Pi_{\rho\gamma}(0) = 0. \quad (5)$$

The node theorem [26] implies that $\Pi_{\rho\rho}(0) = 0$ and therefore $\Pi_{\rho\gamma}(0) = 0$ should be satisfied. These two conditions will be imposed to our model parameters.

The $e^+e^-$ cross section contains an isospin breaking term associated with the $\omega$ meson but also the corresponding one associated with $\phi \to \pi\pi$. However, the corresponding published data [27] are not available in a usable way for fit; fortunately, this effect is concentrated in a narrow region around the $\phi$ mass, and is invisible in the data to be considered. Nevetheless, one could note that the Orsay phase of the $\phi$ meson as well as its branching ratio to $\pi\pi$

are well accounted for within the HLS Model broken in an appropriate way [18].

Before closing this section, let us remark that the $\omega$ contribution has practically no effect somewhat outside the $\omega$ mass region. It is therefore sufficient to treat it as a fixed width Breit–Wigner [16] with accepted values [11] for the $\omega$ mass and width and with a constant phase factor (see (1)). Additionally, we neglect the effects of $\omega-\phi$ mixing by setting $f_{\omega\gamma} = f_{\rho\gamma}/3 = m_\rho^2/3g$. Taking into account the magnitude of this mixing angle [21,22] ($\simeq 3^\circ$ from ideal mixing), this is certainly a safe assumption when fitting the pion form factor.

Therefore, the pion form factor expression used in this paper is (4), supplemented with the $\omega$ contribution as given in (1) with the properties and constraints just described.

## 3 Properties of the one–loop corrections

The loop content of the functions $\Pi_{\rho\rho}(s)$ and $\Pi_{\rho\gamma}(s)$ is determined by the $VPP$, $\gamma PP$, $VVP$ and $\gamma VP$ couplings occuring in the Lagrangian (A10); their expressions are given in Appendix C. All basic $PP$ and $VP$ loops entering the functions $\Pi_{\rho\rho}(s)$ and $\Pi_{\rho\gamma}(s)$ are given by Dispersion Relations and have been computed in closed form in [22]. Their detailed functional structure depends on the usual HLS parameters $g$ and $a$ and, also, on symmetry breaking parameters. These have been fitted several times under various conditions [21–23,18], always providing results consistent with each other; these numerical values will be assumed throughout this paper.

These loops should be subtracted minimally twice ($PP$) or three times ($VP$) from requiring the corresponding Dispersion integrals [22] to be convergent. Therefore, in the full HLS Model (non–anomalous and anomalous sectors), the subtraction polynomials must be at least second degree in $s$ and we can write:

$$\begin{cases} \Pi_{\rho\gamma}(s) = P_\gamma(s) + \overline{\Pi}_{\rho\gamma}(s) \\ \Pi_{\rho\rho}(s) = P_\rho(s) + \overline{\Pi}_{\rho\rho}(s) \end{cases} \tag{6}$$

where the $\overline{\Pi}(s)$ are sums of subtracted loop functions given in [22], and the $P(s)$ are polynomials with real coefficients. We choose to work with second degree polynomials, and then the coefficients to be fitted (or fixed) are defined by:

$$\begin{cases} P_\gamma(s) = d_0 + d_1 s + d_2 s^2 \\ P_\rho(s) = e_0 + e_1 s + e_2 s^2 \end{cases} \tag{7}$$

It is suitable to redefine the ($PP$) $\overline{\Pi}(s)$ functions given in [22] in such a way that they behave like $\mathcal{O}(s^3)$ near the origin. As seen above, it is appropriate to impose $\Pi_{\rho\gamma}(0) = \Pi_{\rho\rho}(0) = 0$ which turns out to fix[2]:

$$e_0 = d_0 = 0.$$

[2] Assuming non–zero $d_0$ and $e_0$, would be *practically* equivalent to releasing any constraint on $m_\rho$ and $f_{\rho\gamma}$ as clear from (4)

It will be shown in the next section that the free parameters of our pion form factor model are only the remaining coefficients of the subtraction polynomials.

A relevant question is whether the polynomials $P(s)$ are really independent of each other or whether the independent polynomials are those associated with the pion and kaon loops contained in the $P(s)$'s. In this case, it is appropriate to check that $P_\gamma(s)$ and $P_\rho(s)$ are not proportional.

Let us discuss here only the non–anomalous sector of the HLS model [17]; information given in the Appendices allows to examine the contributions of the anomalous (FKTUY) sector [20] with analogous conclusions. Using the SU(3) breaking scheme proposed in [24], the piece relevant for the pion form factor can be extracted from (A5) in [24] and can be rewritten in terms of renormalized fields ($K \equiv K_{ren} = \sqrt{z} K_{bare}$, $\pi \equiv \pi_{ren} = \pi_{bare}$):

$$\begin{aligned} &\mathcal{L}_{\rm new} = \\ &\cdots + \frac{iag}{4z}\rho^0 \left[K^- \overleftrightarrow{\partial} K^+ - \bar{K}^0 \overleftrightarrow{\partial} K^0 + 2z\,\pi^- \overleftrightarrow{\partial} \pi^+\right] \\ &+ieA\left[\frac{(z-a/2-a(\ell_V-1)/6)}{z} K^- \overleftrightarrow{\partial} K^+ \right. \\ &\left. -\frac{a(\ell_V-1)}{6z}\bar{K}^0 \overleftrightarrow{\partial} K^0 + (1-a/2)\pi^- \overleftrightarrow{\partial} \pi^+\right] \end{aligned} \tag{8}$$

where $z$ is the SU(3) breaking parameter [3] [28,24]. It should be fixed to $z = [f_K/f_\pi]^2 = 3/2$ in order to recover the correct value of the kaon form factor at $s = 0$. Consistent fits to radiative decay widths of light mesons confirm this value independently [21]. $\ell_V$ is another breaking parameter[4] which has also been fitted [21] using $\omega/\phi$ leptonic decays as $\ell_V = 1.376 \pm 0.031$. Exact SU(3) symmetry corresponds to $z = \ell_V = 1$.

Denoting $\ell_\pi(s)$ and $\ell_K(s)$ the pion and kaon loops amputated from their couplings to external legs (we neglect the mass difference between $K^\pm$ and $K^0$), we derive from Lagrangian (8):

$$\begin{cases} \Pi_{\rho\rho}(s) = g_{\rho\pi\pi}^2\left[\ell_\pi(s) + \dfrac{1}{2z^2}\ell_K(s)\right] \\ \Pi_{\rho\gamma}(s) = g_{\rho\pi\pi}\left[\left(1-\dfrac{a}{2}\right)\ell_\pi(s) + \dfrac{1}{2z^2}\left(z-\dfrac{a}{2}\right)\ell_K(s)\right]. \end{cases} \tag{9}$$

Let us denote $Q_\pi(s)$ and $Q_K(s)$, the subtraction polynomials contained in $\ell_\pi(s)$ and $\ell_K(s)$. Then, these are related with $P_\rho(s)$ and $P_\gamma(s)$ defined above by:

$$\begin{cases} P_\rho(s) = g_{\rho\pi\pi}^2\left[Q_\pi(s) + \dfrac{1}{2z^2}Q_K(s)\right] \\ P_\gamma(s) = g_{\rho\pi\pi}\left[\left(1-\dfrac{a}{2}\right)Q_\pi(s) + \dfrac{1}{2z^2}\left(z-\dfrac{a}{2}\right)Q_K(s)\right]. \end{cases} \tag{10}$$

It is obvious that the single case where $P_\rho(s)$ and $P_\gamma(s)$ are not independent of each other is if $z = 1$ (no breaking

[3] $z$ was also written $1 + c_A$ in [24], referring to the original naming of [28], or $\ell_A$ in [21,22]

[4] We have $\ell_V = (1 + c_V)^2$ in terms of the original breaking parameter of the $\mathcal{L}_V$ term of HLS Lagrangian [28,24]

of SU(3) symmetry); then $e_i = g_{\rho\pi\pi} d_i/(1-a/2)$. Therefore, it is quite legitimate to treat $P_\rho(s)$ and $P_\gamma(s)$ as independent when fitting experimental data.

Some additional remarks are of relevance before closing this section. Within standard VMD ($a=2$), the $\rho$ propagator is still dressed by loop effects as described above. However, one could also expect that no one–loop dressing connects the intermediate photon with the $\rho$ meson and therefore $\Pi_{\rho\gamma}$ would disappear from the form factor (4). The equations just above show that this statement is not true, as:

$$\lim_{a\to 2} \Pi_{\rho\gamma}(s) = \frac{g}{2z^2}(z-1)\ell_K(s) \tag{11}$$

Therefore, an invariant–mass dependent dressing of the $\rho-\gamma$ coupling occurs as consequence of SU(3) symmetry breaking of the HLS model and this statement is valid for all proposed breaking schemes [28,24,29] of the HLS Model[5]. Additionally, it implies that, assuming VMD ($a=2$), the HLS model looses its direct $\gamma\pi^+\pi^-$ coupling, but SU(3) breaking generates direct $\gamma K^+K^-$ and $\gamma K^0\overline{K}^0$ couplings.

A specific character of the HLS model is that it contains a direct coupling of the photon to pseudoscalar pairs and this generates a mass–dependent dressing of the $\gamma-\rho$ transition. However, this property is shared with another identified class of models named VMD1 in [16] (for a review, see [30]). A first such model which illustrates that loop dressing of the $\gamma-\rho$ transition can accomodate pion form factor data is given in [31]; quite recently, the same idea was developed up to a more refined comparison with experimental data up to the $\phi$ mass [32]. We note that it has been explicitly demonstrated that regular VMD and VMD1 are equivalent [25], as one would expect from corresponding fit results [16].

## 4 External phenomenological constraints on $F_\pi(s)$

The HLS model [17] depends basically on only two parameters to be determined experimentally: the universal vector coupling constant $g$ and the parameter $a$ which extends the model beyond the standard VMD assumption ($a=2$). This Lagrangian gives reliable predictions for all hadronic two–body decays of light pseudoscalar provided a suitable breaking scheme is implemented. Previous works [21–23,18] illustrate that the breaking scheme outlined in Appendix A allows to get a successfull account of all experimental data in the realm of the HLS model.

Only a few physics processes can be phenomenologically accounted for without significant symmetry breaking effects, noticeably the pion form factor. Simply using a varying width Breit–Wigner formula[6] for the $\rho$ propagator, the HLS Model can achieve a quite satisfactory description of $F_\pi(s)$ from threshold to the $\phi$ mass [16,15]. This description compares well with other approaches accounting for the Analyticity requirement [1,3,4,31,12,32] or not [16]. Actually, from a phenomenological point of view, the Analyticity assumption for $F_\pi(s)$ gets its full importance only when timelike region data and fits are used to derive predictions outside this region: in the spacelike region or near the chiral point. It was indeed shown in [16] that the behaviour of $F_\pi(s)$ near $s=4m_\pi^2$ predicted from ChPT are well accounted for and that its phase describes quite well the $\delta_1^1(s)$ phase shift up to the $\phi$ mass.

Therefore, even if successfull with $F_\pi(s)$ in isolation, establishing firmly the HLS Model as a consistent framework for physics analysis leads to imposing to the pion form factor to fulfill the parameter constraints of all the rest of the HLS phenomenology [21–23]. As stated several times above, this covers the whole set of radiative and leptonic decays of light mesons. Additionally, it was also shown in [23] that this HLS framework meets all expectations of Extended ChPT [33] concerning decay constants and the mixing angle $\theta_8$. The value derived from our fits for $\theta_0 = -0.05° \pm 0.99°$ did not match well with the leading order ChPT estimate [33] $\theta_0 \simeq -4°$, however, a recent next–to–leading order calculation [34] ($\theta_0 = [-2.5°, +0.5°]$) restores agreement with its phenomenologically extracted value. For thorough discussions on the phenomenological results derived from the broken HLS Model recalled in Appendix A, we refer the reader to [21–23,18].

The parameter values derived from these fits to a very large data set of partial widths are also given in Appendix A and are imposed as numerical constraints to our pion form factor fits. Here we focus on discussing the parameters entering explicitly (4) and the coupling constants affecting the non–anomalous Lagrangian (8): $a$, $g$ and $z$; in the limit of unbroken Isospin Symmetry, the breaking parameter $\ell_V$ drops out from the pion form factor expression.

Pion form factor fits [16,15] give two measurements consistent with each other which can be averaged as $a = 2.35 \pm 0.01$. From a global fit of all radiative and leptonic decays of light meson [21], the best fit value is $a = 2.51 \pm 0.03$. Variants of this model with a mass dependent $\omega-\phi$ mixing angle [22], or accounting for isospin breaking effects [18] give values consistent with this one at never more than 2 $\sigma$.

There is a significant departure between the value of $a$ derived from previous fits to the pion form factor and the value coming from fits to radiative and leptonic decays. As noted in [15], below $s = 1$ GeV$^2$, it could be hard to disentangle completely effects of departures from strict VMD ($a=2$) and effects of resonance tails (namely, the $\rho(1450)$). The global fit to radiative and leptonic decays can be considered as safer from this point of view and then it looks well founded to prefer using $a = 2.51 \pm 0.03$. This turns out to attribute the difference with $a = 2.35$

[5] It is interesting to note that the phase of $F_\pi(s)$ in (4) is given by only the denominator, up to the first inelastic threshold. In the non–anomalous HLS Model this is $K\overline{K}$ and then an imaginary part is generated by a term identical to the one written down in (11) above the $\phi$ mass. If one adds the anomalous sector [20], things become somewhat different as the lowest inelastic threshold becomes $\omega\pi^0$

[6] This does not fulfill the requirement of Analyticity

**Table 1.** Results on the behaviour of $F_\pi(s)$ near $s=0$ from different models, approaches and data sets. Parameters displayed are defined by (12). Entries containing the symbol – are not fitted/given

| | $\lambda_1$ $\mathrm{GeV}^{-2}$ | $\lambda_2$ $\mathrm{GeV}^{-4}$ | $\lambda_3$ $\mathrm{GeV}^{-6}$ |
|---|---|---|---|
| ChPT [37] | $1.88 \pm 0.07$ | $3.85 \pm 0.60$ | $3.0 \pm 1.6$ |
| (without NA7) | $1.88 \pm 0.07$ | $3.85 \pm 0.60$ | $4.1 \pm 1.6$ |
| [39] | $1.93 \pm 0.06$ | $3.90 \pm 0.20$ | $9.70 \pm 0.70$ |
| [1] | $1.86 \pm 0.01$ | $3.60 \pm 0.03$ | – |
| [3] $(\tau)$ | $1.83 \pm 0.05$ | $3.84 \pm 0.03$ | – |
| [3] $(e^+e^-)$ | $1.92 \pm 0.03$ | $3.73 \pm 0.02$ | – |
| [2] $(\tau)$ | $1.89 \pm 0.04$ | $2.1 \pm 1.7$ | $15.2 \pm 5.4$ |
| [2] $(e^+e^-)$ | $1.89 \pm 0.04$ | $6.8 \pm 1.9$ | $-0.7 \pm 6.8$ |

to higher resonance effects not accounted for in the HLS fits in [16,15] and/or to another phenomenon (mass dependent $\rho - \gamma$ coupling).

All fits to the data considered [21–23,18] return $g = 5.65 \pm 0.02$. Finally, fitting the SU(3) breaking parameter $z$ within this data set [21,18] always returned $z = [f_K/f_\pi]^2 = 3/2$, as also expected from $F_K(0) = 1$ [28,24].

If a consistent picture of the HLS phenomenology can be achieved, it implies that these parameters can be fixed at the values corresponding to the best fit in radiative and leptonic decays (values recalled just above). In this case, the only parameters relevant for the $\rho$ meson which can be allowed to vary are the (non–identically zero) coefficients of the subtraction polynomials in (7). Indeed, the HLS Model satisfies a modified KSFR relation which fixes the $\rho$ mass ($m_\rho^2 = ag^2 f_\pi^2$) and $f_{\rho\gamma} = m_\rho^2/g$ in terms of only $a$ and $g$. As we have neglected the $\omega - \phi$ mixing mechanism, $\omega$ is approximated by its ideal component and then $f_{\omega\gamma} = f_{\rho\gamma}/3$ is also fixed.

Therefore, it is a kind of global fit to radiative and leptonic meson decays and to the pion form factor to fit $F_\pi(s)$ by fixing $a$, $g$ and $z$. However, this means that the $\rho$ mass is fixed to $m_\rho = 827 \pm 4$ MeV; using the relation between $g_{\rho\pi\pi}$ and the width, the $\rho$ width would correspond to $\Gamma_\rho \simeq 135$ MeV.

Both values are clearly far from matching expectations [11] and one may wonder how the pion form factor could accomodate such $\rho$ parameters. However, as noted in [22], finite width effects (*i.e*, loop corrections) should restore the consistency. One aim of the present paper is to check and show that all consequences on $\rho$ parameters of the radiative and leptonic decays are indeed accomodated by the pion form factor.

It is also important to point out a couple of subtleties. The $\rho$ mass, as defined by the real part of the propagator $M_\rho$, is highly model dependent [35]. The complex pole, however, is a true invariant, as has been shown for several models [25]. One should also note the difference between $M_\rho$ and the "Higgs–Kibble" mass $m_{HK}$ ($m_\rho^2 = m_{HK}^2 = ag^2 f_\pi^2$) [25] resulting from spontaneous symmetry breaking.

## 5 Probes and data sets

Any fit performed with (4) actually returns an analytic function with some uncertainty on the fit parameters. These fits always optimize the description of data sensitive to only $|F_\pi(s)|$.

A first probe, as for other studies (see [1] for instance), is to compare the phase *predicted* by Arg$[\mathrm{F}_\pi(\mathrm{s})]$ with the most reliable data on the $\delta_1^1(s)$ phase shift [5,8] below $\sqrt{s} \simeq 1$ GeV.

A second probe is to compare numerically the behaviour of this fitted $F_\pi(s)$ near $s=0$ to external sources. These are mostly ChPT predictions [36,37] or approches relying on the inverse amplitude methods [38,39,1,3].

Defining the low energy expansion of $F_\pi(s)$ by:

$$F_\pi(s) = 1 + \lambda_1 s + \lambda_2 s^2 + \lambda_3 s^3 + \cdots \tag{12}$$

the works just quoted find parameter values as given in Table 1; the results of [38] are very close to those displayed and do not quote estimated errors. We also display the results of polynomial fits [2] to timelike data ($\sqrt{s} \leq 0.6$ GeV), fixing the charge radius ($< r_\pi^2 >= 6\lambda_1$) to the value found by the NA7 Collaboration [40].

It is clear from Table 1 that, whatever the method, there is an overall consensus about the value of $\lambda_1$. Even if not as nice, the agreement for the value of $\lambda_2$ looks quite reasonable. The spread of central values for $\lambda_3$ and their accuracies should be however noted. It indicates, at least, some model dependence. It should be noted that published estimates for $\lambda_3$ always go beyond ChPT predictions.

The data sets which basically enter our fits are the former [13] and recent [15] data on $e^+e^- \to \pi^+\pi^-$ together and separately. For convenience, the $\tau$ data [6,7,9] are not considered in the present paper. Additionally, we limit our fits to the region below $s = 1$ GeV$^2$, for reasons to be explained just below.

We also consider the spacelike form factor data of NA7 [40] and of the Fermilab experiment [41] after some check of (fit) consistency with the timelike data. With these data, our fit region extends from $s \simeq -0.25$ to $s \simeq 1$ GeV$^2$.

Finally, we will compare the phase of $F_\pi(s)$ *derived* from fitting $|F_\pi(s)|$ to the phase shift data of [5,8]. These last data sets will be used as probes and not included in the fitted data.

For the time being, we also do not attempt to extend our fit (and/or the HLS Model) to higher $s$ values (namely, above the $\phi$ mass), where effects of $\rho(1450)$ and $\rho(1700)$ have certainly to be accounted for. Extending the HLS Model to such energies is an interesting issue, however, it is not clear whether we would not be going beyond the validity range of the HLS Model which is a low energy model.

## 6 Fitting the pion form factor

In several preliminary studies, we tried examining the behaviour of our fit parameters to the former [13] and recent [15] $e^+e^-$ data. All fit parameters have been found

**Table 2.** Fit results with the HLS Model. Coefficients of the expansion of $F_\pi(s)$ near the origin; notations are those in (12). The first column indicates which are the coupled channels considered in the Model function (4). The number of fitted data points is always 184, the number of free parameters is always 6, including 2 parameters for the $\omega$ contribution. Errors given are derived from the full error matrix computed by MINUIT for the fit parameters

| | $\lambda_1$ GeV$^{-2}$ | $\lambda_2$ GeV$^{-4}$ | $\lambda_3$ GeV$^{-6}$ | $\chi^2/dof$ (Prob) |
|---|---|---|---|---|
| $\pi^+\pi^-$ | $1.899 \pm 0.016$ – | $3.957 \pm 0.017$ – | $10.768 \pm 0.051$ – | $183.6/178 = 1.03$ 36% |
| $\pi^+\pi^- + K\overline{K}$ | $1.899 \pm 0.016$ | $3.958 \pm 0.017$ | $10.772 \pm 0.050$ | $184/178 = 1.03$ 36% |
| $\pi^+\pi^- + \omega\pi^0$ | $1.899 \pm 0.016$ | $3.847 \pm 0.056$ | $12.837 \pm 0.124$ | $173.3/178 = 0.97$ 59% |
| $\pi^+\pi^- + \omega\pi^0 + K\overline{K}$ | $1.896 \pm 0.018$ | $3.848 \pm 0.059$ | $12.841 \pm 0.120$ | $173.6/178 = 0.98$ 58% |
| $\pi^+\pi^- + \omega\pi^0 + K\overline{K} + K^*\overline{K}$ | $1.895 \pm 0.015$ | $3.802 \pm 0.026$ | $15.427 \pm 0.111$ | $170.6/178 = 0.96$ 64% |
| $\pi^+\pi^- + \omega\pi^0 + K\overline{K} + K^*\overline{K} + \rho\eta$ | $1.894 \pm 0.015$ | $3.786 \pm 0.015$ | $23.41 \pm 0.094$ | $169.8/178 = 0.95$ 66% |
| $\pi^+\pi^- + \omega\pi^0 + K\overline{K} + K^*\overline{K} + \rho\eta + \rho\eta'$ | $1.894 \pm 0.014$ | $3.778 \pm 0.012$ | $34.118 \pm 0.046$ | $169.4/178 = 0.95$ 67% |

quite insensitive to any difference, except for the $\omega$ branching fraction to $\pi^+\pi^-$ and the Orsay phase; this particular point will be examined in Sect. 7. Therefore, in this whole section, we consider together the data collected in [13] and [15].

The effect of considering the timelike data [13,15] in isolation and combined with spacelike data [40,41] is more noticeable and amounts to about a $2\sigma$ deviation. Nevertheless, this effect is limited and these data sets contribute to improving the behaviour of the pion form factor in the spacelike region by avoiding to extrapolate without any information. Therefore, for the work reported in this section, we have prefered keeping them in the data set to fit, reestimating the errors correspondingly.

## 6.1 Fit strategies and properties

We report in the following on various strategies used to fit the pion form factor. These differ only by a progressive account of all permitted coupled channels. We stress once again that the number of fitted data points is always the same and that the number of free parameters in the fit is not modified when accounting for more and more coupled channels. We always have the 4 non–zero subtraction parameters defined in (7) which account for the $\rho$ contribution and 2 more parameters to account for the $\omega$ contribution (named $\phi$ and $g_{\omega\pi\pi}$ in (1)).

Qualitatively, all fits give always large correlation (above the 95% level) coefficients $(e_1, e_2)$ and $(d_1, d_2)$. However, the correlation coefficients between both sets is always in the range of 10 to 40%. The parameters defined in (12) are derived by expanding (4) near $s = 0$; when computing errors on the $\lambda_i$, errors and correlations on fitting parameters are taken into account.

On the other hand, the fit qualities associated with subsets of possible coupled channels are displayed as last data column in Table 2. They clearly show, that the fit quality reached below the $\phi$ mass is always good. From examining the evolution of the minimum $\chi^2$, one should note that adding $K\overline{K}$ gives no improvement or no degradation in the model description. In contrast, one could remark the jump in probability when adding the $\omega\pi^0$ channel; indeed, it is a noticeable effect to reduce the minimum $\chi^2$ from $\simeq 184$ to 174 without any additional freedom in the model. This clearly comes from a better account of the pion form factor between the $\omega\pi^0$ threshold and the $\phi$ mass where data are affected by small errors. Instead, the $K\overline{K}$ channels can noticeably affect only a very few data points; their very small effect might be due to the fact that the corresponding loops are numerically negligible when computed very close to threshold.

One could also note that adding the higher $VP$ coupled channels goes on slightly improving the fit quality below the $\phi$ mass; as stressed above, this does not correspond

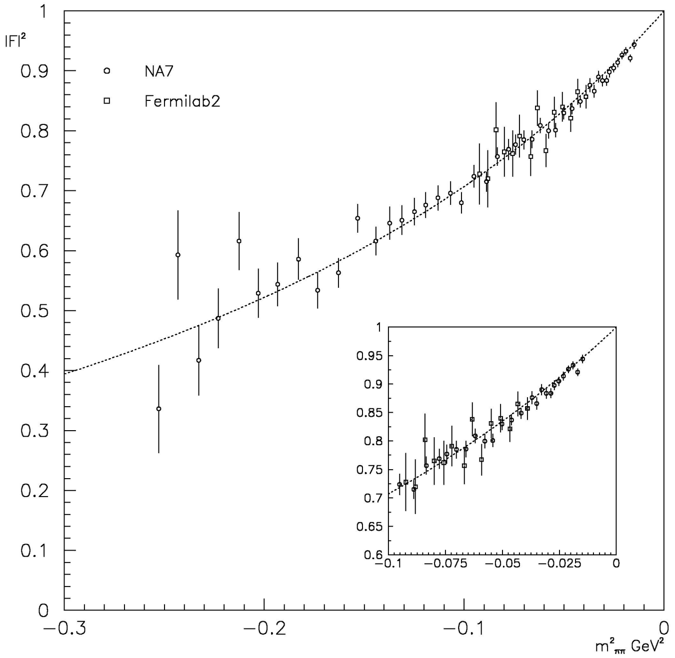


**Fig. 1.** Spacelike data and fit. The data points are from [40] and [41]. The fitting curve has been obtained by considering the $\pi\pi$, $K\overline{K}$ and $\omega\pi^0$ channels. All channels subsets as defined in the body of the text (including $\pi\pi$ alone) give representations hard to distinguish from the one shown

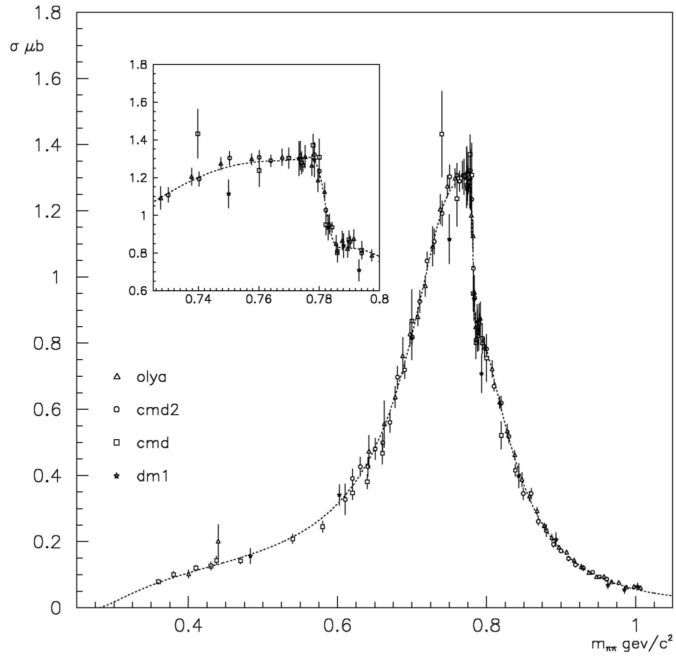


**Fig. 2.** Timelike data and fit. The data points are all subsets from [13–15]. The fitting curve has been obtained by considering the $\pi\pi$, $K\overline{K}$ and $\omega\pi^0$ channels. All channels subsets as defined in the body of the text (including $\pi\pi$ alone) give representations visually identical to the one shown here

to having more freedom in the model. In contrast with the $\omega\pi^0$ channel, effects of these higher threshold loops are modest and can be neglected[7]. One might note, however, that these have more effects on data than the $K\overline{K}$ channels, as clear from Table 2.

Finally, we have attempted fits by removing the function $\Pi_{\rho\gamma}$ from (4), while keeping all parameters fixed by HLS phenomenology at their values obtained from fit to radiative and leptonic decays ($a$, $g$, ...). We never reached a reasonable result. In order to remove this function one clearly needs to release (at least a part of) these constraints in order to allow the fit to converge to a good description of the data[8]. This was not attempted as our aim was to examine the full consequences of having the HLS Model *and* all known numerical constraints coming from its phenomenology. We thus perform a kind of global fit of all relevant decay modes and of the pion form factor together. This exercise, however, tends to indicate that the dressing of the $\rho-\gamma$ transition amplitude is a relevant concept and is requested by the consistency of the pion form factor with the rest of the HLS phenomenology.

Along the same lines, we have attempted fits by fixing the degree of the subtraction polynomials to 1, still fixing the other parameter values ($a$, $g$, $z$). We also never reach a reasonable description of the pion form factor data. Therefore, the model structure looks consistent and motivates the subtraction polynomial degree we choose.

### 6.2 The pion form factor in spacelike and timelike regions

As stated above, whatever the subset of channels considered, the last data column in Table 2 shows that the fit quality is optimum in both the spacelike and timelike $s$ regions considered. Accounting for the three coupled channels $\pi^+\pi^-$, $\omega\pi^0$, $K\overline{K}$ (actually the neutral and charged modes) seems the best motivated coupling scheme for the invariant–mass region from threshold up to the $\phi$ mass. We thus illustrate with this our fit results; visual differences with other channel subsets are tiny.

The fit functions discussed in all this subsection have been obtained from a global fit to all existing timelike data [13–15] and to the spacelike data of [40,41] simultaneously.

In Fig. 1, we display the fitted form factor in the spacelike region and, superimposed, are the data of [40,41]. As expected, the description is quite reasonable.

In Fig. 2, we show the fit in the timelike region superimposed with all existing data [13–15]; in Fig. 3, we have displayed the same fitting curve with only the new data of [15]. Figure 2 shows that the whole energy range is well described, including the region below 600 MeV (measured by CMD). Figure 3 illustrates that the fitting curve derived from fitting all timelike data altogether give also a very good description of the recent CMD–2 data set [15] alone in its full range. It should be noted that, in this

[7] Whether adding $K^*K$ channels and higher threshold channels is appropriate, while neglecting high mass meson contributions or multiparticle loops, can certainly be questioned

[8] The fit quality and results in [4,31,32] clearly proves this statement

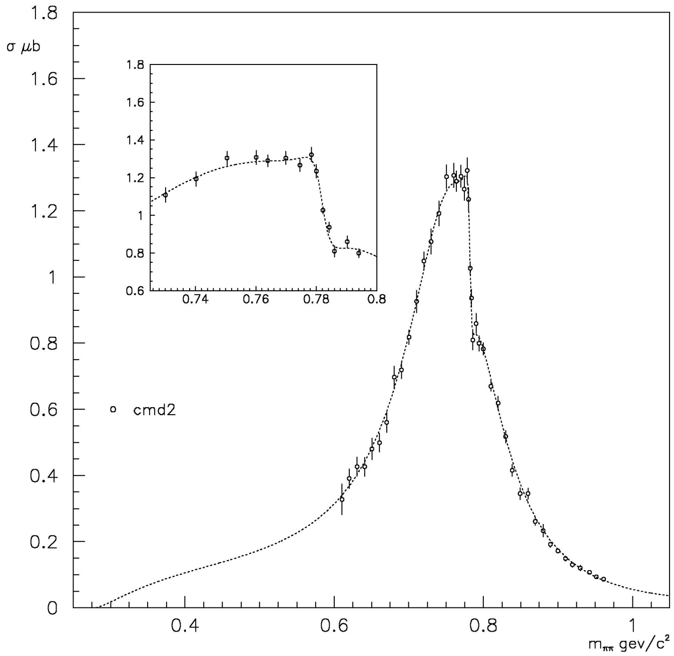


**Fig. 3.** Timelike data and fit. The data points are only from the recent data set collected by the CMD–2 Collaboration [15]. The fitting curve is the same as in Fig. 2 and its numerical coefficients have been determined by a global fit to all available timelike data and to the spacelike data of [40,41]

case, the fitting function corresponds to $\mathrm{Br}(\omega \to \pi^+\pi^-) = 2.12 \pm 0.23\%$; we comment more on this point in Sect. 7.

It should also be remarked that the fitting function being an analytic function of $s$, it is the same function (given in (4) and supplemented with the last term from (1) to account for the $\omega$ mass region) which fits the spacelike and timelike $s$ regions simultaneously.

The noticeable peculiarity of CMD–2 data with respect to the previous data sets is that their systematic errors are smaller than 1% [15]; from what is shown here, one could conclude that the previous data sets, considered altogether, behave globally with small effective systematics. It could also be that the fitting function is analytically enough constrained to be marginally sensitive to systematics.

From what was just discussed, we already know that the data description following from our model is quite reasonable. As clear from Figs. 2 and 3, no need for a $\rho(1450)$ contribution shows up below 1 GeV. This is also illustrated by the fit quality already reached in all cases (see the last data column in Table 2).

As a final remark, one should note that the high value for $m_\rho = 827$ MeV is not inconsistent with the data, provided the model pion form factor is suitably parametrized. We noted already that this mass value derived from HLS phenomenology [21] is very close to the estimate of the vector nonet mass fitted in [4]. This proves, as noted in [22], that it is the loop effects which are responsible for pushing the peak location of the $\rho$ meson (or its pole location) to the customary value attributed to its mass [11].

Now, we focus on comparing refined consequences of our model and fits with numerical predictions concerning the behaviour of the pion form factor at threshold, and the phase of our fitted $F_\pi(s)$.

### 6.3 Pion form factor behaviour at threshold

The results we got concerning the pion form factor behaviour at threshold are gathered in Table 2. They are displayed using the notations of (12). Each line corresponds to a case where a subset of the coupled channels is considered and the size of the coupled channel subset is increased. The second line breaks the obvious rule but is given in order to show that coupling the $K\overline{K}$ has negligible numerical effects on $F_\pi(s)$ and does not improve or degrade the description obtained using only the $\pi\pi$ channel.

A first remark which can be drawn is that $\lambda_1$ (hence, the pion charge radius) is totally insensitive to whatever is added to the $\pi\pi$ channel. For this parameter, our estimate:

$$\lambda_1 = 1.896 \pm [0.018]_{\mathrm{stat}} \pm [0.003]_{syst.}\ \mathrm{GeV}^{-2} \tag{13}$$

is in good agreement with all reported values: ChPT at two–loops result [37], phase method result of [1], resonance ChPT result [4], or the inverse amplitude result of [39] (see Table 1). The systematic error is estimated by considering the variation of the central value of $\lambda_1$ as a function of the subset of coupled channels.

The second coefficient $\lambda_2$ varies only little as function of the number of open channels. However, there is a clear systematic effect: its value decreases slowly when new channels are opened (except for $K\overline{K}$ commented on above). Interestingly, the values we get always match nicely several entries in Table 1. This column in Table 2 leads us to conclude:

$$\lambda_2 = 3.85 \pm [0.06]_{stat.} \pm [0.10]_{syst.}\ \mathrm{GeV}^{-4} \tag{14}$$

where the systematic error is estimated as for $\lambda_1$. This result matches well expectations from Table 1, especially those derived from ChPT.

For the third coefficient $\lambda_3$, the situation is much more embarassing[9]. One should note that $\lambda_3$ depends on the fit parameters ($e_i$ and $d_i$), and also on the third order term of the loops. This third order term is fixed and given by the driving terms of all loops. The only way to change it is to oversubtract the loops and introduce (free) $e_3 s^3$ and $d_3 s^3$ terms in (7) to be fitted and/or fixed. However, the fit quality already reached with fitting data below $\sqrt{s} = 1$ GeV cannot justify to simply increase the model freedom. It thus seems that a reliable estimate of $\lambda_3$ depends on a reliable account of data somewhat above the $\phi$ mass and on other sources of inelasticity generally neglected, like multiparticle loops.

[9] From Table 1 alone, the situation looks already confusing, even by leaving aside the result of [2]. No ChPT estimate for $\lambda_3$ is currently available

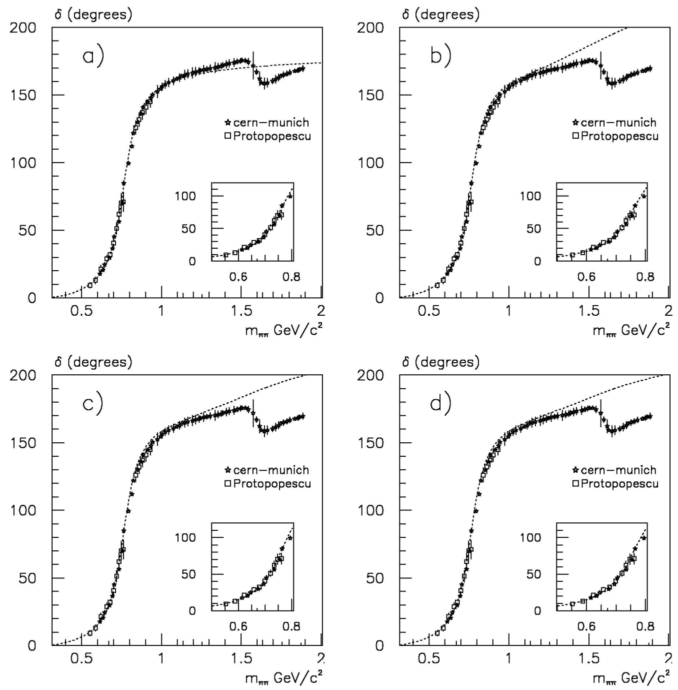


**Fig. 4a–d.** Comparison with the $\pi\pi$ phase shift data of [5] and [8]. The curve plotted is $\mathrm{Arg}[F_\pi(s)]$ with parameters fixed at values corresponding to the best fit of $|F_\pi(s)|$ using all timelike data [13–15] and the spacelike data from [40,41]. In **a**, only the $\pi^+\pi^-$ channel is considered; in **b**, the subset considered is $\pi^+\pi^-$, $\omega\pi^0$ and both $K\overline{K}$ channels. In **c**, the four $K^*K$ channels have been added to the previous channel subset; in **d**, the previous subset is extended so as to include $\rho^0\eta$ and $\rho^0\eta'$. The agreement is perfect up to $\simeq 800$ MeV and good up to $\simeq 1.2$ GeV always

Anyway, one should note first that the first two terms of the chiral expansion of $F_\pi(s)$ are well defined and this is not changed (or spoiled) by adding more and more coupled channels. Secondly, one can assess that the data bounded to $\sqrt{s} \leq 1$ GeV alone look unable to permit a real measurement of $\lambda_3$, as its central value sharply depends on the inelasticity accounted for in the region $\sqrt{s} \geq 1$ GeV. This inelasticity was here represented by high mass channels coupling to the $\rho(770)$ meson, however it could have been anything else (like higher $\rho$ meson contributions). Stated otherwise, without a reasonably good knowledge of (generally neglected) inelasticity effects, the pion form factor cannot provide a reliable estimate of $\lambda_3$.

### 6.4 The phase of $F_\pi(s)$ and phase shift data

As stated above several times, all numerical parameters of the analytic function $F_\pi(s)$ – actually, only its isospin 1 part is relevant here – are derived from fits to data sensitive only to $|F_\pi(s)|$. Therefore $\mathrm{Arg}[F_\pi^{I=1}(s)]$ is a prediction and can be compared with the most precise experimental information on the phase shift $\delta_1^1(s)$ [5,8].

In Fig. 4, we display this comparison using coupling to only $\pi\pi$ (Fig. 4a), then coupling to both $\pi\pi$ and $\omega\pi$ (Fig. 4b); these do not differ from their partners with also the $K\overline{K}$ channels opened. In Fig. 4c, the open channels are all channels up to the 4 contributing $K^*K$ final states; finally, in Fig. 4 d, all possible channels of the full HLS model are considered (the previous subset plus $\rho\eta$ and $\rho\eta'$).

In all cases, the insets show that the low energy region is perfectly predicted up to $m_{\pi\pi} \simeq 800$ MeV, whatever the subset of coupled channels considered.

Using coupling to only $\pi\pi$ (Fig. 4a), the agreement between our prediction and data is perfect up to about 800 MeV and remains very good up to $\sqrt{s} \simeq 1.3$ GeV. Adding $K\overline{K}$ does not modify sensitively this picture.

As soon as one opens the $\omega\pi^0$ channel, the predicted phase starts to diverge almost linearly from the experimental data of [5] from about $m_{\pi\pi} \simeq 1.2$ GeV. Nevertheless, the phase remains perfectly reproduced up to $m_{\pi\pi} \simeq 0.8$ GeV. From about 900 MeV, the predicted phase starts running 2 to 4 degrees above the data of [5]; this effect is systematic but consistent with the data. It is worth remarking that the first inelastic coupled channel in the full HLS model is $\omega\pi^0$ with threshold located at 917 MeV. Therefore, from Watson theorem, one can indeed expect that the phase predicted by the pion form factor and the $\delta_1^1(s)$ phase shift should start diverging[10] at $m_{\pi\pi} = 917$ MeV.

Keeping in mind the words of caution already stated concerning the appropriateness of considering too high threshold mass channels, it is nevertheless interesting to remark a curious effect of the corresponding inelasticity: the quasi–linear rise of the phase above 1.2 GeV which follows from having introduced the coupling to $\omega\pi^0$ is softened more and more, when more (high mass) coupled channels are considered.

We remarked already that our fits to annihilation data below the $\phi$ mass do not exhibit any failure which could be attributed to some neglected $\rho(1450)$ contribution. On the other hand, because we have no guide like the Watson theorem, nothing clear can be stated by observing the higher energy behaviour of the predicted phase when accounting for $VP$ loops. However, the continuation of the annihilation cross section above the $\phi$ mass becomes too large when $VP$ loops are accounted for. Therefore, if fitting some mass region above the $\phi$ meson, beside introducing the $\rho(1450)$ and $\rho(1700)$ mesons, one certainly needs to modify the subtraction scheme by going to higher degree subtraction polynomials. This issue will not be examined any further here.

[10] Actually, it is not that much the divergence between the phase of $F_\pi(s)$ and the $\delta_1^1(s)$ phase shift above 917 MeV which looks appealing. It is rather the agreement between them up to $m_{\pi\pi} \simeq 1.3$ GeV when limiting the subset of coupled channels to $\pi\pi$ and $K\overline{K}$ which could look unphysical. However, examining the elasticity of this wave [5] indicates that the $(I = 1,\ l = 1)$ $\pi\pi$ wave is still elastic at a $\simeq 95\%$ level at this energy. Therefore, nothing conclusive can be derived from this unexpected agreement at relatively large invariant–mass

## 7 The $\omega$ information from fits

We stated in the previous section that there was no noticeable difference between the former annihilation data sets considered together and the new data set, as far as the isospin 1 part of the pion form factor is concerned. However, this does not extend to the $\omega$ parameters accessible from the pion form factor in the timelike region.

We have performed several fits and we report on using the set of all open channels. By closing the high energy ones, one does not change the picture described now.

A fit to all former $\pi\pi$ timelike data [13,14] gives:

$$\mathrm{Br}(\omega \to \pi\pi) = 2.27 \pm 0.35\%,\ \varphi = 106.87^\circ \pm 7.16^\circ \quad (15)$$

with $\chi^2/dof = 63.63/76 = 0.84$ corresponding to a 84% probability. It is close to the accepted value of 2.21±0.30%. On the other hand, the same fit to the new data set of CMD–2 [15] provides:

$$\mathrm{Br}(\omega \to \pi\pi) = 2.01 \pm 0.29\%,\ \varphi = 103.88^\circ \pm 2.91^\circ \quad (16)$$

with $\chi^2/dof = 32.20/37 = 0.87$ corresponding to a 69% probability. This has to be compared with the branching fraction recently published by CMD–2 Collaboration [15] which yields $1.33 \pm 0.25\%$ from their fits. The central values of this result and ours are far apart (however, a $2\sigma$ deviation only); this might also illustrate some model dependence in extracting this information[11]. We have nevertheless checked our extracted values by considering several subsets of open channels with never more than $\simeq 0.3\ \sigma$ fluctuations.

## 8 Conclusion

This study leads us to several conclusions. First, an expression for the pion form factor can be derived from the HLS Model which fulfills all expected analyticity requirements. In this approach, the $\rho - \gamma$ transition amplitude becomes invariant–mass dependent and several two–body channels couple to $\pi\pi$; this arises as a natural feature of the full HLS Lagrangian. Among these additional couplings, the $\omega\pi^0$ channel plays an interesting role as it is lower in mass than the $K\overline{K}$ channels, more commonly accounted for.

The derived description of timelike and spacelike experimental data is found consistent with all the rest of the HLS phenomenology which was examined in detail elsewhere. This includes also the HLS–KSFR relation which defines a $\rho$ mass of $\simeq 827$ MeV perfectly accepted by the pion form factor data. In the present modelling, the fit parameters are essentially subtraction constants (for the $\rho$) and isospin symmetry violation parameters (for the $\omega$).

Among the additional channels to be considered, a special role is devoted to the $\omega\pi^0$ channel which affects fit qualities by a significant jump in probability. This reflects a better account of the invariant–mass region from the $\omega\pi^0$ threshold to the $\phi$ mass. In contrast, the $K\overline{K}$ channels are found to provide no improvement and, even, no change at all in fit qualities below the $\phi$ mass.

The model is fitted on data only sensitive to $|F_\pi(s)|$. The phase of $F_\pi(s)$ is thus a prediction which can be compared with the data on the $\delta_1^1(s)$ phase shift. It is found to match perfectly these from threshold to about the $\rho$ mass. The agreement remains very good up to $\simeq 1$ GeV and a little above independently of the channel subset considered. All this matches well expectations from the Watson theorem. We detect no difficulty which would lead to include a $\rho(1450)$ contribution in order to improve the fit quality below the $\phi$ mass.

The terms of order $s$ and $s^2$ of $F_\pi(s)$ at the chiral point are found highly stable, with little or no sensitivity to the inelasticity accounted for. They are found in fairly well agreement with all known accepted values. The term of order $s^3$ is found instead to depend sharply on the inelasticity accounted for; one may question the possibility to extract this information reliably using only experimental data below the $\phi$ mass.

The $\omega$ branching fraction to $\pi\pi$ is found smaller in the data set recently collected by the CMD–2 Collaboration than in the former data sets (2.01±0.29% instead of 2.27±0.35%), however not as much as previously claimed (1.33±0.25%).

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# Appendix

## A The lagrangian model

We outline in this Appendix the Lagrangian Model used in order to derive the pion form factor. The construction of the unbroken HLS Lagrangian can be found in [17] and, specifically in [20] for its anomalous part. The breaking scheme we use has been constructed in several steps starting for the basic BKY mechanism as expressed in [43]. In [24], we have first defined a way to break SU(3) flavor symmetry which preserves hermiticity of the non–anomalous HLS Lagrangian. Breaking of nonet symmetry was first introduced in [21] and examined in [23] in connection with the Extended ChPT framework [33]. In [21,22] some features especially related with the $K^*$ (anomalous) sector were considered.

[11] It should be remarked that our fit of the data collected in [13] gives a result close to the published fit of OLAY and CMD data (namely $2.30 \pm 0.5\%$). For this fit, [13] was taking into account the coupling to $\omega\pi^0$ channel in the way proposed by [42]. Fitting the former data in [13] as done now with the new data gives instead $2.00 \pm 0.34\%$ [15]

The non–anomalous HLS Lagrangian can be written [17]:

$$\mathcal{L}_{\rm NA} = \mathcal{L}_{\rm A} + a\mathcal{L}_{\rm V} \tag{A1}$$

where $a$ is a parameter specific of the HLS Model; the traditional formulation of vector dominance corresponds to setting $a = 2$. At lowest order, after gauging for both electromagnetism and the hidden local symmetry and after breaking the SU(3) flavor symmetry, we have [24]:

$$\begin{aligned}\mathcal{L}_{\rm A} &= {\rm Tr}[\partial P\ X_A \partial P\ X_A \\ &\quad +2ieA\ (P\ Q - Q\ P)\ X_A\ \partial P\ X_A] \\ \mathcal{L}_{\rm V} &= {\rm Tr}\left[f_\pi^2\ [(gV - eAQ)\ X_V]^2 \right.\\ &\quad \left. +i(gV - eAQ)\ X_V\ (\partial P\ P - P\ \partial P)\ X_V\right].\end{aligned} \tag{A2}$$

In these expressions, $A$ denotes the electromagnetic field, $V$ and $P$ denote the matrices of resp. the vector and pseudoscalar (bare) fields; we normalize these matrices as defined in [24]. The quark charge matrix is $e\ Q$ where $Q = {\rm Diag}(2/3, -1/3, -1/3)$ and $g$ is the universal vector coupling [17].

$X_A$ and $X_V$ are symmetry breaking matrices originally introduced in [43]:

$$\begin{aligned}X_A &= {\rm Diag}(1, 1, \sqrt{z}) \\ X_V &= {\rm Diag}(1, 1, \sqrt{\ell_V})\end{aligned} \tag{A3}$$

where $z = [f_K/f_\pi]^2$ and $\ell_V$ is to be determined from data. The expanded form of the Lagrangian $\mathcal{L}_{\rm NA}$ is given in [24]. SU(3) Symmetry breaking breaks the canonical form of this Lagrangian; the transformation which restores this property is:

$$P' = X_A\ P\ X_A \tag{A4}$$

which gives the renormalized field matrix $P'$ in terms of the bare one $P$. Nonet Symmetry breaking is introduced by adding to the Lagrangian in (A1), a piece derived from determinant terms of the field matrix [44]; this is given by [23]:

$$\mathcal{L}_{\rm tH} = \frac{1}{2}\lambda \partial_\mu \eta_0 \partial^\mu \eta_0 - \frac{1}{2}\mu_0^2 \eta_0^2 \tag{A5}$$

where $\eta_0$ denotes the bare pseudoscalar singlet field. The $\lambda$ term of this Lagrangian piece also contributes to modifying the kinetic energy term in a non–canonical way. The full transformation which restores canonicity after breaking of both SU(3) and Nonet Symmetries is derived in [23]. It has been shown in this reference that the full canonical tranformation can be simplified at leading order in breaking parameters $z$ and $\lambda$. This turns out to use (A4) while redefining the renormalized field matrix $P'$:

$$P' = P_8' + xP_0' \tag{A6}$$

where the subscripts 8 and 0 stand for, respectively, the octet and singlet parts of the pseudoscalar field matrix. $x$ becomes the Nonet Symmetry breaking parameter($x \simeq 1/\sqrt{1+\lambda}$) and has been fitted [21,23] from radiative decays of light mesons to $x \simeq 0.90$, indicating a 10% breaking of Nonet Symmetry.

In our study of the pion form factor, we are interested in accounting for the coupling of the $\rho$ meson to vector and pseudoscalar mesons. These coupling are anomalous and are provided by the following piece of the (FKTUY) Lagrangian [20]:

$$\mathcal{L}_{\rm FKTUY} = C\epsilon^{\mu\nu\alpha\beta}{\rm Tr}[\partial_\mu V_\nu \partial_\alpha V_\beta P] \tag{A7}$$

in terms of the bare field matrices. This Lagrangian (reexpressed in terms of renormalized fields using (A4) and (A6)) allows for a good description of all modes except those involving the charged $K^*$ [21,22]. In order to get a successful description of all partial widths, the Lagrangian in (A7) has to be broken. This can be achieved by combining the breaking (BGP) procedure defined in [29] together with the replacement[12] $V \to X_T V X_T$. Then, the broken anomalous piece above becomes [21,22]:

$$\begin{aligned}\mathcal{L}_{\rm FKTUY} = C\epsilon^{\mu\nu\alpha\beta}{\rm Tr}&\left[\left(X_T \partial_\mu V_\nu X_T^{-1}\right)\right.\\ &\left.\times \left(X_T^{-1}\partial_\alpha V_\beta X_T\right)\ P\right]\end{aligned} \tag{A8}$$

This Lagrangian has been examined in full detail in [21, 22] and its expanded form is given in [21]. All modes not involving the $K^*$ mesons are strictly unaffected by this breaking procedure, particularly the coupling $(\rho\omega\pi)$. Instead modes involving the $K^*$ sector are affected. However, from a numerical point of view, couplings to neutral $K^*$'s yield a negligible change while couplings to charged $K^*$'s are deeply modified. It is interesting to note that this procedure allows to recover the expression obtained by Dillon and Morpurgo [45] for $K^*$ coupling constants and coincide with the corresponding formulae derived in the non–relativistic quark model (NRQM). The coefficient in (A7) and (A8) is [20] $C = -3g^2/4\pi^2 f_\pi$ and the breaking matrix writes [21,22]:

$$X_T = {\rm Diag}(1, 1, \sqrt{\ell_T}) \tag{A9}$$

Concerning the pion form factor this breaking mechanism plays a very minor role, as it does not affect the coupling $g_{\rho\omega\pi}$ and the $\rho K^* K$ couplings have little influence below the $\phi$ mass.

Therefore the full HLS broken Lagrangian Model is:

$$\mathcal{L}_{\rm HLS} = \mathcal{L}_{\rm NA} + \mathcal{L}_{\rm tH} + \mathcal{L}_{\rm FKTUY} \tag{A10}$$

and depends on a few parameters. Fits to radiative and leptonic decays of light mesons [21–23] confirm the prediction [43,24] for $z$ ($z = [f_K/f_\pi]^2 = 3/2$ with a remarkable precision). They also permit to fix the other breaking parameters: $\ell_V = 1.38 \pm 0.03$, $\ell_T = 1.19 \pm 0.06$ and $x = 0.90 \pm 0.02$. Additionally, these decays allow to determine the fundamental parameters of the HLS model: $a = 2.51 \pm 0.03$ and $g = 5.65 \pm 0.02$.

Stated otherwise, all physics parameters are fixed by radiative and leptonic decays and not only the symmetry breaking ones. Therefore, in order to impose the consistency of the HLS Model and check its appropriateness to

[12] This replacement resembles a renormalization of the vector field matrix which is absent in the HLS model

the whole low energy phenomenology, the freedom available when fitting the pion form factor is very small; this only deals with the subtraction constants values and the subset of open channels. All coupling constants relevant for the pion form factor within the HLS model are listed in Sect. C below.

## B Structure of $\Pi_{\rho\rho}(s)$ and $\Pi_{\rho\gamma}(s)$

The equation defining the pion form factor is given by (4) with, in addition, an isospin violating term as given in (1)– the $\omega$ contribution. Besides standard quantities, it involves the $\gamma \to \rho$ transition amplitude $\Pi_{\rho\gamma}(s)$ and the $\rho$ meson self–mass $\Pi_{\rho\rho}(s)$.

$\Pi_{\rho\gamma}(s)$ and $\Pi_{\rho\rho}(s)$ are constructed from $PP$ and $VP$ loops. All loops considered here should be understood amputated from their coupling constants to external ($\gamma$ and $\rho$) lines. As stated in the body of the text, multiparticle loops (not present in the basic HLS Lagrangian) are not considered.

Within the non–anomalous HLS Lagrangian, the photon and the $\rho$ meson couple both to $\pi^+\pi^-$, $K^+K^-$ and $K^0\overline{K}^0$; couplings to $\pi^+\pi^-$, $K^+K^-$ exist as soon as $a \neq 2$ and are modified by symmetry breaking effects. Instead the coupling to $K^0\overline{K}^0$ is generated by SU(3) breaking of the $\mathcal{L}_V$ HLS Lagrangian [17,24]. Neglecting the kaon mass splitting, this gives rise to two loop functions (pion and kaon loops), given in closed form in [22] and named $\ell_\pi(s)$ and $\ell_K(s)$ in the body of the text.

When taking into account the anomalous (FKTUY) sector [20] of the HLS model, other intermediate states have to be considered; first, we have $\omega\pi^0$, $\rho^0\eta$ and $\rho^0\eta'$, neglecting the $\omega\phi$ mixing. This gives rise to three additional $VP$ loops, also given in closed form in [22]; they will be denoted resp. $\ell_\omega(s)$, $\ell_\eta(s)$ and $\ell_{\eta'}(s)$. The couplings to $K^{*+}K^-$, $K^{*-}K^+$, $K^{*0}\overline{K}^0$, $\overline{K}^{*0}K^0$ give rise to the same amputated loop denoted $\ell_{K^*}(s)$ if one neglects the mass splitting generated by isospin breaking.

These basic loops come within $\Pi_{\rho\rho}(s)$ multiplied each by the square of their coupling constants to $\rho$, while in $\Pi_{\rho\gamma}(s)$, they are multiplied by the product of their coupling constants to $\rho$ and to the photon.

Whatever the (sub)set of loops effectively taken into account, it should be stressed that this does not modify the freedom of our model, as soon as one chooses to subtract these functions three times; to a large extent, these two information can be disconnected, as one can choose externally the number of subtractions to be performed, and there is no reason why the number of subtractions should be minimal.

Actually, increasing the subset of coupled channels turns out only to add definite functions with given couplings determined numerically elsewhere by fits to radiative and leptonic decays. These couplings will be listed below.

Taking into account the effects of $\ell_\eta(s)$, $\ell_{\eta'}(s)$ and $\ell_{K^*}(s)$ below the $\phi$ mass might be discussed, while neglecting the tails of the $\rho(1450)$ and $\rho(1700)$ contributions or multiparticle loop effects. However, considering besides the pion loop, the kaon loop with threshold at $\sqrt{s} \simeq 1$ GeV, while neglecting the $\omega\pi^0$ with threshold at $\sqrt{s} = 0.917$ GeV seems unjustified. Therefore, we can cautiously consider that fit results with $\pi^+\pi^-$, $K\overline{K}$ and $\omega\pi^0$ should be more relevant than their analogues with only $\pi^+\pi^-$ and $K\overline{K}$.

## C Coupling constants

From the Lagrangian piece written in (8), we can derive:

$$\begin{cases} g_{\rho\pi\pi} = \dfrac{ag}{2} \; , \; g_{\gamma\pi\pi} = (1-\dfrac{a}{2})e \\ g_{\rho K^+K^-} = \dfrac{ag}{4z} \; , \; g_{\gamma K^+K^-} = (z-\dfrac{a}{2}-b)\dfrac{e}{z} \\ g_{\rho K^0\overline{K}^0} = -\dfrac{ag}{4z} \; , \; g_{\gamma K^0\overline{K}^0} = -\dfrac{be}{z} \end{cases} \tag{C1}$$

where $b = a(\ell_V - 1)/6$. From our previous works [21–23], the symmetry breaking parameters are all fixed as stated just above.

From the anomalous Lagrangian pieces $VVP$ and $VP\gamma$ given in [21], setting:

$$C_\omega = -\frac{3g^2}{8\pi^2 f_\pi} \; , \; G_\omega = -\frac{3g}{8\pi^2 f_\pi} \tag{C2}$$

we get:

$$\begin{cases} g_{\rho^0\omega\pi^0} = C_\omega \; , \; g_{\gamma\omega\pi^0} = G_\omega \, e \\ g_{\rho^0 K^{*\pm}K^\mp} = \sqrt{\dfrac{\ell_T}{z}}\dfrac{C_\omega}{2} \; , \\ \quad g_{\gamma K^{*\pm}K^\mp} = \sqrt{\dfrac{\ell_T}{z}}\left(2-\dfrac{1}{\ell_T}\right)\dfrac{G_\omega}{3}\, e \\ g_{\rho^0 K^{*0}K^0} = -\sqrt{\dfrac{\ell_T}{z}}\dfrac{C_\omega}{2} \; , \\ \quad g_{\gamma K^{*0}K^0} = -\sqrt{\dfrac{\ell_T}{z}}\left(1+\dfrac{1}{\ell_T}\right)\dfrac{G_\omega}{3}\, e \end{cases} \tag{C3}$$

with [21,22] $\ell_T = 1.19 \pm 0.06$ being an additional breaking parameter which has been introduced independently by [45].

Defining the physical $\eta/\eta'$ fields in terms of singlet and octet fields $\eta_0$ and $\eta_8$ has been shown [23] to meet all requirements of Extended ChPT [33], including now [34] the extracted value for $\theta_0$. One could also work in the strange/non–strange field basis [46], but the correspondence can be done [47] and lead to substantially the same numerical results. Thus, defining the pseudoscalar mixing angle by:

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix} = \begin{bmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{bmatrix} \begin{bmatrix} \eta_8 \\ \eta_0 \end{bmatrix} \tag{C4}$$

and setting $\theta_P = \theta_{ideal} + \delta_P$, we have:

$$\left\{\begin{array}{ll} g_{\rho^0\rho^0\eta} &= \dfrac{C_\omega}{6}\left[\sqrt{2}(1-x)\cos\delta_P-(1+2x)\sin\delta_P\right] \\ g_{\rho^0\rho^0\eta'} &= \dfrac{C_\omega}{6}\left[\sqrt{2}(1-x)\sin\delta_P+(1+2x)\cos\delta_P\right] \\ g_{\gamma\rho^0\eta} &= \dfrac{e\ G_\omega}{3}\left[\sqrt{2}(1-x)\cos\delta_P-(1+2x)\sin\delta_P\right] \\ g_{\gamma\rho^0\eta'} &= \dfrac{e\ G_\omega}{3}\left[\sqrt{2}(1-x)\sin\delta_P+(1+2x)\cos\delta_P\right] \end{array}\right. \tag{C5}$$

where [23] $\theta_P = -10.32° \pm 0.20°$. $x$ is a parameter accounting for Nonet Symmetry breaking (no breaking corresponding to $x = 1$). It was fitted as independent parameter [21] to $x = 0.917 \pm 0.017$ with a large correlation coefficient [23] $(\theta_P, x)$. In [23], it was shown that the observed quasi–vanishing of $\theta_0$ implies that

$$\theta_P \simeq \sqrt{2}\frac{(1-z)}{2+z}\ x. \tag{C6}$$

This is numerically well fulfilled and leads to a fit quality identical to those obtained in [21] where this condition was not requested; this however lessens significantly correlations among fit parameters. This corresponds to $x = 0.901 \pm 0.018$, which is the value choosen for the present work.

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